independent interpretation of static observations. Minshall [61M1], Taylor and Rice [63T2], and Ivanov et al. [63I1] observed precursor-wave amplitudes that decreased with increasing distance of propagation. Duvall [64D2], Taylor [65T2], and Ahrens and Duvall [66A2] demonstrated that such behavior could be accounted for by stress relaxation. Decompression-wave profiles are strongly influenced by elastic-plastic response and early measurements on metals by Curran [63C1], Erkman and Christensen [67E1], Fuller and Price [65F1, 69F1], Barker [68B2], Kusubov and van Thiel [69K5], and others demonstrated that the hydrodynamic approximation did not adequately describe this wave in the range of compressive stresses to a few tens of GPa. These early phenomenological investigations, which are briefly summarized by Herrmann [69H1, 74H1, 76H3] and Murri et al. [74M3] are sufficient to demonstrate that neither the hydrodynamic approximation nor the ideal elastic-plastic model adequately describes the material response. Most recent work centers on interpretation of observations in terms of microscopically-based viscoplastic response models and on studies of shear strength of the compressed material, although several especially comprehensive continuum-mechanical investigations of metals have been conducted by Christman and coworkers [70C1, 71C3, 71C4, 72C2, 72C3, 72I1].

Much of the early work reflects the confrontation of inadequate theoretical models and experimental techniques with a complicated physical phenomenon. Viewed in retrospect, the most useful outcome of this work, besides realization and acceptance of the complexity of the problem, was the development of improved experimental techniques and computational methods that permit numerical simulation of experiments in terms of realistic material models.

3.3.2. Theory

When an elastic body is compressed in uniaxial strain, the pressure and magnitude of the shear stress increase by approximately proportional amounts. This proportional increase ceases when the shear stress reaches some critical level called the elastic limit or yield stress. In the simplest case the shear stress remains constant at this limiting value during further compression, while the pressure continues to increase as before. This behavior, shown in fig. 3.4a, is called ideal elastic-plastic response. The steep low-stress part of the heavily-drawn curve represents the elastic response of the material as described by eq. $(3.4)_1$. The limit of the elastic range is marked HEL. The portion of the curve beyond this limit represents the combined effects of isotropic elastic compression, some elastic response to shear, and the inelastic shear response. If an elastic-perfectly plastic body compressed to the state A is allowed to expand, the expansion process comprises an elastic expansion AB followed by plastic flow to C. These processes are discussed in detail by various authors, including Wood [52W1], Morland [59M1], Fowles [61F2], Zel'dovich and Raizer [66Z1], and Cristescu [67C4].

Because plastic deformations involve hysteretic effects, it is customary to express the theory in terms of equations relating incremental changes in stress and strain, or the rates of these quantities, rather than their current values. In uniaxial strain, $\boldsymbol{S}$ takes the diagonal form discussed in section 2.1, but is decomposed into two parts so that $\boldsymbol{S} = \boldsymbol{S}^{e} + \boldsymbol{S}^{p}$, where $\boldsymbol{S}^{e}$ represents an elastic and $\boldsymbol{S}^{p}$ an inelastic contribution to the strain. The associated strain rate tensor is defined to be $\dot{\boldsymbol{S}} = \dot{\boldsymbol{S}}^{e} + \dot{\boldsymbol{S}}^{p}$. The symmetry of the problem (for isotropic materials) suggests that each of these parts of the strain rate be symmetric about the X_1 axis but the separate parts need not be, and in fact are not, uniaxial. The plastic strain corresponds essentially to a displacement of the lattice from one to another equivalent arrangement of the atoms, so it does not involve any volume change. This interpretation of plastic strain also means that the stress rate is to be calculated from the elastic part of the strain

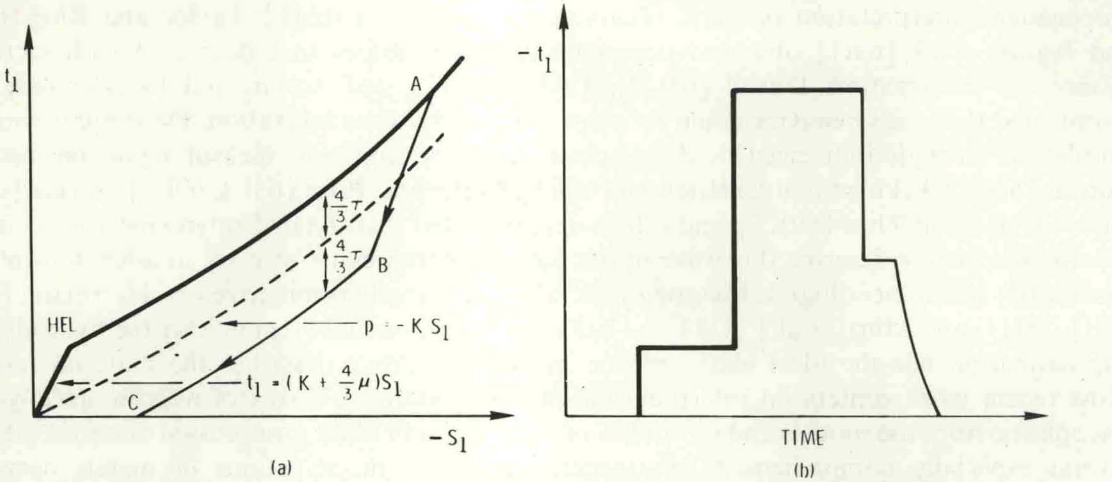

Fig. 3.4. Ideal elastic-plastic response. Part (a) of the figure shows the Hugoniot curve (as a heavy line), with the part of the stress attributable to the pressure response shown as the dashed line. The decompression process originating at a state (A) follows the lightly drawn path (ABC). The stress history that results at a point within a slab from short-duration application of stress at its boundary is shown in part (b) of the figure.

rate, and the result can be written in the form

$$\dot{t}_1 = (K + \tfrac{4}{3}\mu)\dot{S}_1 - \tfrac{8}{3}\mu\dot{\gamma}^{\mathrm{p}}, \qquad \dot{t}_2 = (K - \tfrac{2}{3}\mu)\dot{S}_1 + \tfrac{4}{3}\mu\dot{\gamma}^{\mathrm{p}}, \tag{3.7}$$

where $\dot{\gamma}^{\mathrm{p}}$ is the rate of plastic shear strain, given by $\dot{\gamma}^{\mathrm{p}} = (\dot{S}_1^{\mathrm{p}} - \dot{S}_2^{\mathrm{p}}) = \frac{3}{4}\dot{S}_1^{\mathrm{p}}$. The ideal elastic-plastic response can be recovered from equations (3.7), but our interest here lies with the more complicated responses.

The variety and complexity of elastic-plastic phenomena make the development of a mechanical model of observed behavior a very difficult task even though, in essence, all that has to be done is represent the dependence of the plastic shear strain rate $\dot{\gamma}^{\mathrm{p}}$ in eq. $(3.7)_1$ in terms of other variables of the problem.

The first breakthrough in this area came with the application of elementary concepts of dislocation mechanics. According to this model the plastic shear rate $\dot{\gamma}^{\mathrm{p}}$ on a given slip system is related to the dislocation motion by the Orowan relation

$$2\dot{\gamma}^{\mathrm{p}} = bNV_{\mathrm{d}}, \tag{3.8}$$

where b is the length of Burgers' vector (a constant of the order of the lattice constant), N is the total length of mobile dislocation line in a unit volume of material, and V_{d} is the average velocity of the mobile dislocations. The dislocation density N depends in a complicated way (usually through a differential equation) on the entire history of the deformation in a neighborhood of the point in question, and V_{d} is a function of the resolved shear stress, τ, i.e., the component of $\boldsymbol{t}$ on the slip plane in the direction of the slip. The regime of elastic response is introduced into the theory by setting $V_{\mathrm{d}}(\tau)$ to zero when $|\tau| < \tau_0$ for some limiting shear stress τ_0. Strain hardening is taken into account by allowing τ_0 to increase as plastic strain is accumulated. The dependence of V_{d} on τ is sufficiently strong that it is not unreasonable to assume that all slip occurs on planes lying at 45° to the X_1 axis. When this is done, the shear rate given by eq. (3.8) can be substituted directly